A Simple and General Analytical Expression for Calculating Crosstalk in Multiwaveguide Directional Couplers Using Finite Differences Method

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ABSTRACT

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1. INTRODUCTION

Directional couplers are, in general, composed of two parallel waveguides (WG) placed close enough to give rise to optical coupling. DCs are considered as an elementary parts for optical switching/modulation, optical power splitter/combiners [1] and electro-optic multiplexers [2]. In this regards, many researchers studied DC [3-6], many of them reported the way of minimizing CT caused by the coupling effect in tapered regions of the reversed delta beta parallel DC switches by using phase control [7]. By contrast, others presented a novel kind of terahertz DC which can achieve a lower absorption loss based on photonic crystals [8] and others considered the CT as an optical scatterings caused by devices defects [9]. Some CT can be treated as a noise sources on the impulse signals which have the same effect regardless of the signal strength. If the signal strength drops too much, the effect of noise increases [10]. It is also reported on CT of 2DC and 3DC with outer fed, when an incident wave is splitted into each normal mode unevenly [11]. When signals from one channel are crossed over another channel, they become noisy in the other channel. This can lead to serious effects on the signal-to-noise ratio and hence on the error rate of the system. However, the most important problems in optical communication systems are unwanted CT, which is one of the factors that results in increasing the bit error rate. Crosstalk is quoted as the loss in dB between the input level of the signal and its (unwanted) signal strength in the adjacent channel. Also, It is possible to allow significantly more crosstalk in a modulator than in switch (all depending on the application) [12]. Channel crosstalk considerations are analogous to the polarization splitter design described in [13].

As our knowledge, there is no general mathematical expression in the literatures for determining the CT in multichannel DC, hence, in this paper, an analytical expression for calculating CT in multiwaveguide directional couplers (MWG DC) via controlling the power to be input from a chosen channel was derived.

2. EXPERIMENTAL

Fig.1 illustrates a typical integrated multiwaveguide directional coupler (MWG DC). The refractive index profile of cladding and guiding regions are $n_1$ and $n_2$, respectively with waveguide width $d$ and separation $S$.

For simplicity, we will analyze 2WG DC with incident of field $E_1$ on WG1 and $E_2$ on WG2. The incident power are distributed between the two even and odd normal modes $\psi_1$ and $\psi_2$ respectively. Therefore, the local fields can be expressed in terms of $\psi_1(x)$ and $\psi_2(x)$ as follows:

$$E_1(x) = a_{11}\psi_1(x) + a_{12}\psi_2(x) \quad (1a)$$
$$E_2(x) = a_{21}\psi_1(x) + a_{22}\psi_2(x) \quad (1b)$$

![Fig.1: Refractive index profile for integrated MWG DC.](image)
Where $\psi_i$ and $\psi_j$ are orthonormal functions with coefficients $a_{ij}$ represent the contribution of the normal mode $\psi_j$ in the local field $E_i$.

$$[E] = [a][\psi]$$

(2)

Or

$$E_i = \sum_{j=1}^{2} a_{ij} \psi_j \quad , \quad i = 1, 2$$

(3)

Multiplying Eq. (3) by $\psi_k$ and integrating over $x$, using the orthonormal properties of the normal modes yields:

$$E_{T_i}(z) = \sum_{j=1}^{2} a_{ij} \psi_j e^{-j\beta_j z} \quad , \quad \text{for} \quad i = 1, 2$$

(5)

Where $\beta_j$ is the propagation constant of the normal mode $j$. The $CT_1$ due to the first input WG after propagation a distance equal to the coupling length ($z = L_c$) is:

$$CT_1 = \left| \int E_i E_{T_i}(z = L_c) dx \right|^2$$

(6)

And $CT_2$ due to the second input channel is

$$CT_2 = \left| \int E_2 E_{T_2}(z = L_c) dx \right|^2$$

(7)

This leads to a CT expression in terms of $a_{ij}$'s

$$CT_1 = \left| a_{11} e^{-j\beta_1 z} + a_{12} e^{-j\beta_2 z} \right|^2$$

(8)

$$CT_2 = \left| a_{21} e^{-j\beta_1 z} + a_{22} e^{-j\beta_2 z} \right|^2$$

(9)

For 3WG DC, local modes can be written as a superposition of normal modes $\psi_1, \psi_2$ and $\psi_3$ as follows

$$E_1 = a_{11}\psi_1 + a_{12}\psi_2 + a_{13}\psi_3$$

(10a)

$$E_2 = a_{21}\psi_1 + a_{22}\psi_2 + a_{23}\psi_3$$

(10b)

$$E_3 = a_{31}\psi_1 + a_{32}\psi_2 + a_{33}\psi_3$$

(10c)
For 3WG DC, one can find the field evolution as follows

\[ E_{T_1}(z) = a_{11} \psi_1 e^{-j\beta_1 z} + a_{12} \psi_2 e^{-j\beta_2 z} + a_{13} \psi_3 e^{-j\beta_3 z} \]  
\( (11a) \)

\[ E_{T_2}(z) = a_{21} \psi_1 e^{-j\beta_1 z} + a_{22} \psi_2 e^{-j\beta_2 z} + a_{23} \psi_3 e^{-j\beta_3 z} \]  
\( (11b) \)

\[ E_{T_3}(z) = a_{31} \psi_1 e^{-j\beta_1 z} + a_{32} \psi_2 e^{-j\beta_2 z} + a_{33} \psi_3 e^{-j\beta_3 z} \]  
\( (11c) \)

If the input in the first, second and third channel respectively, the corresponding CT are:

\[ \text{CT}_1 = \left| \int E_1 E_{T_1}(z = L_c) dx \right|^2 \]  
\( (12a) \)

\[ \text{CT}_2 = \left| \int E_2 E_{T_2}(z = L_c/2) dx \right|^2 \]  
\( (12b) \)

\[ \text{CT}_3 = \left| \int E_3 E_{T_3}(z = L_c) dx \right|^2 \]  
\( (12c) \)

Where CT_3 is due to the third input channel, and L_c is the coupling length. If the power was launched from channel two, we used z = L_c/2 in calculating CT_2 since the power is totally coupled to the adjacent channels after propagating this distant. Substituting Eqs. (10) and (11) into Eq.(12) leads to a CT expression in terms of the coefficients a_{ij} as follows:

\[ \text{CT}_1 = \left| a_{11}^2 e^{-j\beta_1 z} + a_{12}^2 e^{-j\beta_2 z} + a_{13}^2 e^{-j\beta_3 z} \right|^2, \quad z = L_c \]  
\( (13a) \)

\[ \text{CT}_2 = \left| a_{21}^2 e^{-j\beta_1 z} + a_{22}^2 e^{-j\beta_2 z} + a_{23}^2 e^{-j\beta_3 z} \right|^2, \quad z = L_c/2 \]  
\( (13b) \)

\[ \text{CT}_3 = \left| a_{31}^2 e^{-j\beta_1 z} + a_{32}^2 e^{-j\beta_2 z} + a_{33}^2 e^{-j\beta_3 z} \right|^2, \quad z = L_c \]  
\( (13c) \)

In conclusion, a simple and general analytical expression for determining CT in MWG DC for a chosen \( i^{th} \) input channel after travelling a distant L can be expressed through the following formula:

\[ \text{CT}_i = \left| \sum_{j=1}^{N} a_{ij}^2 \psi_j e^{-j\beta_j L} \right|^2, \text{ for } i=1,2,3..., N \]  
\( (14) \)

where \( L = \begin{cases} \frac{L_c}{2}, & \text{if } N \text{ is odd} \\ \frac{N+1}{2} \end{cases} \)

The evolution of total electric field due to the input power from channel i is

\[ E_{T_i}(z) = \sum_{j=1}^{N} a_{ij} \psi_j e^{-j\beta_j z} \]  
\( i = 1, 2, ..., N \)  
\( (15) \)
3. RESULTS AND DISCUSSION

In this section, numerical results of the CT is presented in order to ensure the validity of the proposed expression which expressed by Eq. (14). A MATLAB codes was developed to simulate numerically MWG DC’s using FDM. Local and normal mode field profiles and their corresponding propagation constants are determined for a dielectric slab and MWG DC’s, respectively. Selection of the input power to the entered from the required WG was done by using Eq. (15).

The application of this procedure is used to describe the evolution of the mode intensity profile along its propagating direction for multi planar WG DC with $n_1=2.20$, $n_2=2.2025$, $S=6\quad m$ and $d=5\quad m$ as shown in figures (2-5). The above WG DC parameters are selected to support only a single mode at $\lambda = 1.3\, \mu m$. Moreover, single-mode fibers were designed to have zero dispersion at this wavelength.

We used FDM to calculate the eigenvalues for TE polarization $\lambda = 1.3\, \mu m$, which is commonly used in telecommunications, of the following structures: for 2WG DC, the structure support two normal modes with effective refractive indices of 2.20142351 and 2.20130385, and support three normal modes with 2.20148022, 2.20145835 and 2.20129317 for 3WG DC, four modes with 2.20151453, 2.20145835, 2.20137929 and 2.20130436 for 4WG DC and five modes with 2.20152060, 2.20148057, 2.20142045, 2.20135214 and 2.20129503 for 5WG DC.

Figures (2-5) show that the light wave was coupled from one channel to another with ability of controlling the input power to be entered from appropriate channel which is important in calculating CT.

In order to ensure the validity of the general expression Eq. (14), CT in MWG DC was investigated as an example, as shown in figures (6-9). Results showed that CT can be controlled by adjusting $L_c$ and the gab separation. The coupling length $L_c$ was decreased rapidly because of strong coupling between the waveguide cores as the gap separation was decreased by several micrometers. Conversely, the wider separation gap, $s$, gets the smaller CT which is desirable in multiplexers but it required larger DC length to complete power translation between different channels.

Kim, et. al., [1] used FD method and BPM to study CT for just two and three WG DC. Therefore, our Results are compared them for 2 and 3WG DC and showed an excellent agreement.
Fig. 3: Power distribution in three WG DC if the input is from (a) First WG, (b) second WG and (c) Third WG.

Fig. 4: Power distribution in four WG DC if the input is from (a) First WG, (b) second WG, (c) Third WG and (d) Fourth WG.
Fig. 5 Power distribution in five WG DC if the input is from (a) First WG, (b) second WG, (c) Third WG, (d) Fourth WG and (e) Fifth WG.

Fig. 6 (a) Coupling length and CT in 2WG DC as a function of S and (b) CT comparison with Kim et. al. [1].
Fig. 7 (a) Coupling length and CT in 3WG DC as a function of S for input power from First WG and (b) CT comparison with Kim et al. [1].

Fig. 8 Coupling length and CT in 4WG DC as a function of S for input power from (a) First WG and (b) Second WG.
Fig.9: Coupling length and CT in 5DC as a function of S for input power from (a) First WG, (b) Second WG and (c) Third WG

4. REFERENCES
The paper presented a general analytical expression for calculating CT with different input channel for MWG DC. Simulation results of this analytical expression are based on the FDM and when compared with the available literatures, it gives an excellent agreement. Results showed that the proposed procedure is significantly accurate and useful especially for MWG DC, dense wavelength division multiplexers based on DCs, optical switches and Mach-Zehnder interferometers

5. REFERENCES